PMETp-w

Program to calculate potential evapotranspiration (ET_p) at **monthly**, **daily**, **hourly** or **shorter** periods using micrometeorological data and the parameters described below. The calculation procedure is based on the modified Penman-Monteith method as described by Allen et al. (1998).

MICROMETEOROLOGICAL DATA

Input variables for computing ET_p at **DAILY** or **MONTHLY** time step are: Month, Day, Maximum air temperature (ºC), Minimum air temperature (ºC), Maximum relative humidity (%), Minimum relative humidity (%), Solar radiation (W/m^2), and Wind speed (m/s):

Month	Day	Tmax(C)	Tmin(C)	RHmax(%)	RHmin(%)	$R_g(W/m^2)$	u(m/s)
7	11	27.84	13.19	81.62	37.26	317.01	1.920
7	12	28.08	21.36	81.81	36.92	247.81	1.980
7	13	25.60	13.37	81.98	44.68	333.64	2.580
7	14	17.22	11.63	89.88	56.88	339.95	3.220
7	15	14.71	10.84	89.88	76.04	166.44	4.160
7	16	13.22	10.72	89.87	70.44	155.64	4.320

Input variables for computing ET_p at **HOURLY** or **SHORTER** time steps are: Month, Day, Air temperature (ºC), Relative humidity (%), Solar radiation (W/m^2), and Wind speed (m/s)

Month	Day	Hour	T(C)	RH(%)	$R_g(W/m^2)$	u(m/s)
3	11	14.00	16.39	37.71	877.7	4.26
3	11	14.15	16.29	38.52	739.5	4.95
3	11	14.30	15.94	39.26	193.5	3.33
3	11	14.45	15.28	40.64	34.0	2.76
3	11	15.00	14.32	41.05	20.5	3.29

PARAMETERS

Weather station information

- Altitude (m asl)
- Latitude (rad)
- Longitude (rad)
- Longitude of the Standard Meridian (rad)
- Difference to solar time (h)

Data information

- Normal or leap year
- Time step (min)
- Height of wind speed measurements (m)
- Height of humidity measurements (m)

Crop information

- Crop height (m)
- Albedo
- Active Leaf Area Index
- Leaf stomata resistance (s/m)

For calculating leaf stomata resistance:

- Minimum leaf stomata resistance (s/m)
- Shape parameter for calculating as a function of global radiation.

Other parameters

- Coefficients ac and bc for calculating cloudiness factor (ac+bc=1)
  Default values: ac= 1.35, bc= -0.35

- Coefficients a1 and b1 for calculating net emissivity
  Default values: a1= 0.34, b1= -0.139

ETp CALCULATION

Air temperature (ºC)

For daily time step:

$$T = \frac{Tmax + Tmin}{2}$$

Tmax is the daily maximum air temperature (ºC)
Tmin is the daily minimum air temperature (ºC)

Relative humidity (ºC)

For daily time step:

$$RH = \frac{RHmax + RHmin}{2}$$

RHmax is the daily maximum relative humidity (%)
RHmin is the daily minimum relative humidity (%)

Saturated vapor pressure (Pa)

For daily time step:

$$e_s = \frac{e_s(Tmax) + e_s(Tmin)}{2}$$

e_s(Tmin) is the saturated vapor pressure (Pa) at Tmin
e_s(Tmax) is the saturated vapor pressure (Pa) at Tmax
T is the air temperature (ºC)

For hourly or shorter time steps:

$$e_s(T) = 611 \exp\left(\frac{17.27T}{237.3 + T}\right)$$

Actual vapor pressure (Pa)

For daily time step:

$$e_a = \frac{e_s\left(Tmin\right) \cdot RHmax + e_s\left(Tmax\right) \cdot RHmin}{200}$$

For hourly or shorter time steps:

$$e_a = e_s \cdot \frac{RH}{100}$$

e_s(Tmin) is the saturated vapor pressure (Pa) at Tmin
e_s(Tmax) is the saturated vapor pressure (Pa) at Tmax
RHmax is the daily maximum relative humidity (%)
RHmin is the daily minimum relative humidity (%)
e_s is the saturated vapor pressure (Pa)
RH is the relative humidity (%)

Vapor pressure deficit (Pa)

$$VPD = e_s - e_a$$

e_s and e_a are the saturated and actual vapor pressures (Pa), respectively

Coefficient for Boundary-layer resistance (-)

$$r_{a\_c} = \frac{\ln\left[\left(z_u - d\right)/z_{om}\right]\ln\left[\left(z_h - d\right)/z_{oh}\right]}{k^2}$$

For open water evaporation (r_c=0):

$$r_{a\_c} = 4.72\ln\left[\left(z_u - d\right)/z_{om}\right]\ln\left[\left(z_h - d\right)/z_{oh}\right]$$

z_u is the height of wind speed measurements (m)
z_h is the height of humidity measurements (m)
h is the average height of the canopy (m)
d is the zero plane displacement (m): d=2h/3
z_{om} is the momentum roughness length (m): z_{om}=0.123h
z_{oh} is the vapor roughness length (m): z_{oh}=0.1z_{om}
k is the von Karman's constant: k= 0.41

Boundary-layer resistance (s/m)

$$r_a = \frac{r_{a\_c}}{u_z}$$

r_{a_c} is the coefficient for boundary-layer resistance (-)
u_z is wind speed measurements (m/s)
(if $u<0.5$ m/s $\Rightarrow r_a = r_{a\_c}/0.5$ m/s)

For open water evaporation (r_c=0):

$$r_a = \frac{r_{a\_c}}{1+0.536u_z}$$

Atmospheric Pressure (Pa)

$$P_o = 101300\left(\frac{273.15+T}{273.15+T+\kappa Z_e}\right)^{\frac{g}{R_a\kappa}}$$

T is the air temperature (ºC)
g is the acceleration due to gravity (m/s^2): g= 9.81
R_a is the gas constant of air (m^{2}s^{-2}K^{-1}): $R_a \approx 287$
κ is the adiabatic lapse rate (K/m): κ= 0.0065
Z_e is the measurement site altitude (m asl)

Latent heat of vaporization (J/kg)

$$\lambda = 2.501\times10^6 - 2361T$$

T is the air temperature (ºC)

Psychrometric constant (Pa/ºC)

$$\gamma = \frac{c_p P_o}{0.622\lambda}$$

c_p is the specific heat of moist air (J kg^{-1}K^{-1}): c_p= 1013
P_o is the atmospheric pressure (Pa)
λ is the latent heat of vaporization (J/kg)

Slope of the vapor pressure function (Pa/ºC)

$$\Delta = \frac{4098e_s}{\left(237.3+T\right)^2}$$

e_s is the saturated vapor pressure (Pa)
T is the air temperature (ºC)

Virtual temperature (K)

$$T_{vk} = \left(273.15+T\right)\left(1+0.378\frac{e_a}{P_o}\right)$$

T is the air temperature (ºC)
e_a is the actual vapor pressure (Pa)
P_o is the atmospheric pressure (Pa)

Air density (kg/m^3)

$$\rho_a = \frac{P_o}{R_a T_{vk}}$$

P_o is the atmospheric pressure (Pa)
R_a is the gas constant of air (m^{2}s^{-2}K^{-1}): $R_a \approx 287$
T_{vk} is the virtual temperature (K)

Net radiation (W/m^2)

$$R_n = \left(1-\alpha\right)R_g - R_{nl}$$

R_n is the net radiation (W m^{-2})
α is the albedo
R_g is the solar global radiation (W m^{-2})
R_{nl} is the isothermal long-wave radiation (W m^{-2})

Soil heat flux (W/m^2)

For daily time step:

$$G = 0$$

For hourly or shorter time steps:

$$G = 0.1R_n \text{ when } R_n > 0$$

$$G = 0.5R_n \text{ when } R_n < 0$$

R_n is the net radiation (W m^{-2})

Canopy or bulk surface resistance (s/m)

$$r_c = \frac{R_{leaf}}{LAI_{act}}$$

R_{leaf}: leaf stomata resistance (s/m)
LAI_{act} is the active Leaf Area Index

Modified Psychrometric constant (Pa/ºC)

$$\gamma^* = \gamma\left(1 + {r_c}\Big/{r_a}\right)$$

γ is the psychrometric constant (Pa/ºC)
r_c is the canopy surface resistance (s/m)
r_a is the boundary-layer aerodynamic resistance (s/m)

Radiation term (mm)

$$rET = \frac{\Delta(R_n - G)}{\lambda(\Delta + \gamma^*)} t_s \cdot 60$$

$$rET = 0 \quad \text{when} \quad rET < 0$$

Δ is the slope of the vapor pressure function (Pa/ºC)
R_n is the net radiation (W/m^2)
G is the soil heat flux (W/m^2)
λ is the latent heat of vaporization (J/kg)
γ* is the modified psychrometric constant (Pa/ºC)
t_s is the data time step (minutes)

Aerodynamic term (mm)

$$aET = \frac{\rho_a c_p VPD}{\lambda(\Delta + \gamma^*) r_a} t_s \cdot 60$$

ρ_a is the air density (kg/m^3)
c_p is the specific heat of moist air (J kg^{-1}K^{-1}): c_p= 1013
VPD is the vapor pressure deficit (Pa)
λ is the latent heat of vaporization (J/kg)
Δ is the slope of the vapor pressure function (Pa/ºC)
γ* is the modified psychrometric constant (Pa/ºC)
t_s is the data time step (minutes)

Potential Evapotranspiration (mm)

$$ET_p = rET + aET$$

rET is the evapotranspiration radiation term (mm)
aET is the evapotranspiration aerodynamic term (mm)

PARAMETER VALUES FOR THE REFERENCES CROP

- Crop height: 0.12 m
- Albedo: 0.23
- Active Leaf Area Index: 2.88/2 =1.44
- Leaf stomata resistance (s/m):

  For daily time step: R_{leaf}=100.8 s/m when R_n>0, R_{leaf}=1008 s/m when R_n<0

  For hourly or shorter time steps: R_{leaf}=72 s/m when R_n>0, R_{leaf}=288 s/m when R_n<0

COMPUTING THE ISOTHERMAL LONG WAVE RADIATION

Clear sky total solar global radiation (W m^{-2})

$$R_{so} = (0.75 + 2 \times 10^{-5} Z_e) R_a \frac{60}{0.0036 t_s}$$

Z_e is the measurement site altitude (m asl)
R_a is the extraterrestrial solar radiation (MJ m^{-2})

Apparent net emissivity for clear sky conditions

$$\varepsilon' = a1 + b1\sqrt{e_a \times 10^{-3}}$$

a1 and b1 are coefficients with default values: a1= 0.34, b1= -0.139
e_a is the actual vapor pressure (Pa)

Cloudiness factor

$$f = ac\frac{R_g}{R_{so}} + bc \qquad \text{where } 0.3 \le \frac{R_g}{R_{so}} \le 1$$

$$0 \le f \le 1$$

$$f < 0 \quad \text{when } f = f_o$$

ac and bc are coefficients (ac+bc=1) with default values: ac= 1.35, bc= -0.35
R_g is the solar global radiation (W m^{-2})
R_{so} is the clear sky solar global radiation (W m^{-2})
f_o is the cloudiness factor of the previous period (default value 0.6)

Net long wave radiation (W m^{-2})

For daily time step:

$$R_{nl} = f\varepsilon'\sigma \frac{(273.15+Tmin)^4 + (273.15+Tmax)^4}{2}$$

For hourly or shorter time steps:

$$R_{nl} = f\varepsilon'\sigma(273.15+T)^4$$

f is the cloudiness factor
ε' is the net emissivity for clear sky conditions
σ is the Stefan-Boltzman ct. (W m^{-2} K^{-4}): σ= 5.67x10^{-8}
Tmax is the daily maximum air temperature (ºC)
Tmin is the daily minimum air temperature (ºC)
T is the air temperature (ºC)

COMPUTING THE EXTRATERRESTRIAL SOLAR RADIATION

Solar declination (rad)

$$\delta = \frac{23.45\pi}{180}\sin\left[\frac{2\pi}{N_{year}}\left(J-80.75\right)\right]$$

J is the day of year: $1 \le J \le N_{year}$
N_{year} is the number of days in year (366 for leap years)

Inverse relative distance Earth-Sun

$$d_r = 1 + 0.033\cos\left(\frac{2\pi}{N_{year}}J\right)$$

J is the day of year: $1 \le J \le N_{year}$
N_{year} is the number of days in year (366 for leap years)

Seasonal correction for solar time (h)

$$S_c = 0.1645\sin(2b) - 0.1255\cos(b) - 0.025\sin(b)$$

$$\text{with} \quad b = \frac{2\pi}{364}(J-81)$$

J is the day of year: $1 \le J \le N_{year}$

Note: For monthly time step J corresponds to day 15 of the corresponding month.

Time of solar noon (h)

$$h_o = 12 - (L_z - L_m)\frac{180}{15\pi} - S_c$$

L_z is the longitude of the local time meridian (rad):
for Greenwich, L_z= 0
L_m is the longitude of the measurement site (rad)
S_c is the seasonal correction for solar time (h)

Clock time (h)

$$h = d_t - d_{ls} - \frac{t_s}{2\cdot 60}$$

d_t is the local standard time (h)
d_{ls} is the difference to solar time (h)
t_s is the data time step (minutes)

Solar time angle at midpoint of the period (rad)

$$\omega = \frac{\pi}{12}\left(h - h_o\right)$$

h is the clock time (h)
h_o is the time of solar noon (h)

Sunset hour angle (rad)

$$\omega_s = \arccos\left[-\tan(\varphi)\tan(\delta)\right]$$

φ is the latitude of the measurement site (rad)
δ is the solar declination (rad)

Sinus of the solar altitude angle (rad)

For daily time step:

$$\sin\theta = \omega_s\sin(\varphi)\sin(\delta) + \cos(\varphi)\cos(\delta)\sin(\omega_s)$$

For hourly or shorter time steps

$$\sin\theta = \left(\omega_2 - \omega_1\right)\sin(\varphi)\sin(\delta) +$$

ω_s is the sunset hour angle (rad)
φ is the latitude of the measurement site (rad)
δ is the solar declination (rad)
ω_1 is solar time angle (rad) at the beginning of period
ω_2 is solar time angle (rad) at the end of the period

$$+\cos(\varphi)\cos(\delta)\left[\sin(\omega_2)-\sin(\omega_1)\right]$$

$$\sin\theta = 0 \quad \text{when } \omega_2 = \omega_1$$

$$\omega_1 = \omega - 0.5\frac{\pi}{12}\frac{t_s}{60} \qquad \omega_2 = \omega + 0.5\frac{\pi}{12}\frac{t_s}{60}$$

$$\omega_1 = -\omega_s \text{ when } \omega_1 < -\omega_s$$
$$\omega_2 = -\omega_s \text{ when } \omega_2 < -\omega_s$$
$$\omega_1 = \omega_s \text{ when } \omega_1 > \omega_s$$
$$\omega_2 = \omega_s \text{ when } \omega_2 > \omega_s$$

Extraterrestrial solar irradiance falling on a surface parallel to the ground (MJ m^{-2})

$$R_a = \frac{t_s G_{SC} d_r \sin\theta}{\pi}$$

t_s is the data time step (minutes)
G_{SC} is the solar constant (MJ m^{-2} min^{-1}): G_{sc}= 0.08202
d_r is the inverse relative distance Earth-Sun
$\sin\theta$ is sinus of the solar altitude angle (rad)

COMPUTING R_{leaf} AS A FUNCTION OF GLOBAL RADIATION

According to Jarvis (1976) and Stewart (1988) the leaf stomata resistance ($R_{leaf\_min} \leq R_{leaf} \leq R_{leaf\_max}$= 4100 s/m) can be estimated as the product of the minimum resistance, R_{leaf_min} (i.e. maximum conductance) corresponding to optimal conditions, times various stress functions (interacting without synergy), which vary with temperature, relative humidity, global radiation, leaf water potential (and thus also soil water potential), and ambient CO_2 concentration. When none of the factors is limiting, $R_{leaf} = R_{leaf\_min}$.

To compute the stomatal resistance here, only those stress functions where considered, which accounts for the response of stomata to radiation, temperature and humidity:

$$R_{leaf} = R_{leaf\_\min} F_1^{-1} F_2^{-1} F_3^{-1}$$

The influence of solar radiation can be expressed in terms of a hyperbolic function of the form (Stewart, 1988):

$$F_1 = \frac{R_g\left(1 + c / R_{g\_\max}\right)}{c + R_g}, R_g > 0$$

$$F_1 = \frac{R_{leaf\_min}}{R_{leaf\_max}}, R_g = 0$$

where R_g is global solar radiation with R_{g_max}= 1300 W m^{-2} and c is a shape parameter. Stewart (1988) derived a mean value of c=100, showing that this equation is not sensitive to a ±20% variation of this parameter.

Stomata open as ambient temperature increases up to an optimum value (T_{opt}), after which they start closing. Thus, the response of stomata to temperature is given by:

$$F_2 = 1 - k\left[(273.15 + T) - T_{opt}\right]^2$$

where T is ambient temperature (ºC). Following Lhomme et al. (1998), T_{opt}=298 K and k= 0.0016 K^{-2}.

According to Leuning (1995) and Daily et al. (2004), the effect of air humidity on stomatal resistance can be computed as:

$$F_3 = \frac{1}{1 + VPD/VPD_x}$$

where VPD is the vapor pressure deficit (kg kg^{-1}) and VPD_x is 0.0077 kg kg^{-1}.

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